Jarnik's Problem

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Proved there can be at most two integer lattice points on the arc if you restrict your attention to a short enough arc of a circle

… how “short” is short?

Claim

1. If an arc is of length less than L(r), then it contains at most 2 integer lattice points.

1. L(r) is equivalent to the shortest arc that contains 3 integer lattice points.

Finding β

Arc length s = r * θ

$$s = \alpha r^{\beta}$$

$\log_r s = \beta = \log(r * \theta)/\log(r)$

What is the lower bound for β?

Program

```
c=[]
u=[]
p=10
P=[]
for r=1:100000
    v=[]
    for x=0:r
        a=sqrt(r^2-x^2)
        if a-floor(a)==0
            v=[v;atan(a/x)]
        end
    end
    a=length(v)
    L=[]
    if a==2
        L=[pi]
    else
        for i=1:a-2
            L=[L, v(i,1)-v(i+2,1)]
        end
```

Example Data

RADIUS	β
1.0000	Inf
2.0000	2.6515
3.0000	2.0420
4.0000	1.8257
5.0000	0.9531
6.0000	1.6389
7.0000	1.5883
8.0000	1.5505
9.0000	1.5210
10.0000	0.9672
11.0000	1.4774
12.0000	1.4607
13.0000	1.0632
14.0000	1.4338
15.0000	0.9721
16.0000	1.4129
17.0000	1.0274
18.0000	1.3960
19.0000	1.3888

Pattern in the Triples

β

Radius

Pattern in the Triples (Color Coded)

β

Radius

16004 Radii Graph

β

Radius

Minimum β Data

β

β=0.4377

Radius

Finding the Minimum β

Circumscribed Circle Area Theorem
Area of Triangle = (a * b * c) / (4 * r)

Relationship between *s* and θ?

Arc length > but close to chord length
$a \leq (r * \theta)/2$
$b \leq (r * \theta)/2$
Chord $c \leq r * \theta$

Finding the Minimum β

- Plug in a, b, c for A= (a * b * c) / (4 * r)

A = r2θ3

1<A< r2θ3

1<r2θ3

r * 1 < r3θ3

r < s3 so s > $\sqrt[3]{r}$ so s3 > r1/3

β > 1/3